TEACHING & LEARNING RESEARCH INITIATIVE
NĀU I WHATU TE KĀKAHU, HE TĀNIKO TAKU

Making Mathematical Thinking Visible

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Introduction

Mathematics can be viewed as complex structures comprising intricate and profoundly useful webs of relationships. Students can experience mathematics in this way when working on mathematical tasks such as problem solving, modelling, investigations or problem posing. Yet these kinds of tasks are seldom implemented in many upper-secondary and tertiary mathematics classrooms in New Zealand, and are sometimes dismissed by students and educators with the remark, "These are fun problems, but where's the maths?"

We chose to view this situation as a problem of communication. Could we develop effective ways of communicating the messy, complex mathematical structures students create and manipulate while working on such tasks? Doing so might help students, teachers, and other stakeholders to recognise the mathematical value of these mathematical tasks and integrate them more meaningfully into mathematical instruction in New Zealand.

Aims

Our goal was to develop ways of reporting to students, teachers and other stakeholders, the complex mathematical activity students engage in while exploring mathematical structures. We focused on a specific class of mathematical tasks that we call *risky mathematising tasks*, in which students are required to create an original (to them) mathematical product through mathematising a challenging, complex and novel situation.

We aimed to develop forms of reporting that highlighted the complex mathematising students engaged in during these tasks, while retaining and respecting the holistic (structural) nature of such mathematical activity.

Why is this research important?

Alternative mathematical tasks like the *risky mathematising tasks* we developed are well known for emphasising a range of important mathematical competencies that traditional sets of exercises do not, such as mathematical communication, collaboration, creativity and critical thinking. Yet if they are not also seen as offering students powerful experiences with mathematical *content*, they are likely to be relegated to one-off rainy-day activities rather than integrated meaningfully into the mathematics curriculum.

Key findings

We developed a practical format for reporting to students their complex mathematical activity. This format involves two sessions:

(1) *Mathematical reflection session*, which aims to enhance students' awareness of their own mathematical activity by encouraging them to listen and respond to the conversations of mathematically skilled observers reflecting on their work;

(2) *Mathematical comparison session,* which seeks to deepen students' critical appreciation of their own mathematical activity by comparing it to others' mathematical approaches to the same task.

These reporting sessions were developed through 26 cycles of design, testing and revision, with 52 students and six educators from diverse instructional settings in senior secondary and tertiary institutions. The design of the reporting sessions draws on a diverse range of theoretical and practical influences, including narrative inquiry, philosophy of mathematics, and counselling approaches.

Implications for practice

The two-part format of reporting that we have designed offers a novel way of giving feedback to students on their complex mathematical activity to encourage reflection and self-awareness. The *mathematical reflection session* provides opportunities for students to 'eavesdrop' on observers reflecting about their mathematical activity in a way that reduces the potential for confrontation, conflict and negative responses to criticism, and is modeled after a process from family therapy called 'reflecting teams'. The accompanying *mathematical comparison session* encourages students to further deepen their mathematical sensitivity by critically examining their own work in comparison to other students' solutions to the same problem.

This format has also been effective in raising educators' awareness of the rich and complex mathematical activity students engage in during *risky mathematising tasks*. Rather than presenting educators with a checklist of learning outcomes, the reporting format first sensitises them to the kinds of mathematics they themselves engage with during these tasks, which they then invoke as they notice and appreciate the range of mathematical possibilities that students can experience through the tasks.

Our partners

We partnered with six mathematics teaching practitioners from six different instructional settings: Year 12 and 13 mathematics classes from two secondary schools; pre-degree bridging courses in algebra from two different foundation studies programmes; an undergraduate mathematics courses for non-mathematics majors and an undergraduate course for students majoring in the mathematical sciences. We also partnered with a counselling supervisor to develop our practical skills as mathematical reflectors.

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