GUIDE #5

LEARNING IN UNDERGRADUATE MATHEMATICS: THE OUTCOME SPECTRUM (LUMOS) "HOW TO" GUIDES

# Monitor the development of mathematical communication

BILL BARTON AND BARBARA KENSINGTON-MILLER

# A series of "How to" guides

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**LEARNING IN UNDERGRADUATE MATHEMATICS: THE OUTCOME SPECTRUM (LUMOS).**

**“HOW TO” GUIDE #5:**
**MONITOR THE DEVELOPMENT OF MATHEMATICAL COMMUNICATION**

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# Contents

# Mathematical Communication and why we should observe it

Our survey of lecturers and stakeholders in undergraduate mathematics education revealed that mathematical communication was a strongly desired outcome of studying mathematics at tertiary level. It was a desirable attribute both for students entering graduate study in the mathematical sciences and related subjects, and for those entering the workforce who might be expected to use mathematical skills. A simple knowledge of mathematics was not sufficient—mathematics graduates also need to know how to communicate their subject to other mathematically educated people, and also to those with less mathematical experience who are part of workplace teams.

For example, debate still rages over whether a lack of adequate communication by mathematicians in the finance world about the proper use of their mathematical tools was responsible for the 2007/8 financial crisis. The subsequent development by a Cambridge mathematician of a code of ethics for those in this field indicates at least some responsibility to communicate effectively.

So, mathematical communication is valued, but we rarely explicitly address it in our courses, nor do we have the means to determine whether our students develop competency in it. This guide is a contribution to the latter deficiency.

But first, what exactly do we mean by mathematical communication? How can we know whether our courses meet this demand?

Written communication includes:

- Mathematical notation in coherent, structured form.
- Prose descriptions, explanations or arguments.
- Graphs and diagrams to supplement mathematical notation or prose.

Oral communication includes:

- Listening and understanding others' statements and arguments, including seeking more information and prompting further discussion.
- Making mathematical statements, descriptions, and arguments at a level appropriate for the audience.
- Responding appropriately to questions.

A considerable literature exists on this topic, mostly, but not all, relating to secondary school mathematics. For an overview see Morgan, Craig, Schuette & Wagner (2014). For a mathematician's take on the topic, see Folland, 2010.

# Overview of recommended observation technique

## Written Communication

Written communication can be observed using standard examinations, tests, or assignments, with a small modification. The modification is to include in the examination (or test or assignment) one or two questions that require some form of written response in addition to showing mathematical working. Two forms need to be elicited: explanations or descriptions that require a few prose sentences, and graphs or diagrams that supplement mathematical notation or prose. See Appendix 1 for some sample questions.

In addition to these general communicative skills, the ability to mathematically record justifications and arguments, as well as formal proofs, is a separate skill. Standard questions asking for such justification or proof are sufficient tools; however, one further requirement is necessary: students must be told that their work will be evaluated for its communicative clarity and correctness. If this is not done, many students will focus only on ensuring each step is mentioned in order to get full marks, but not worry about the links between steps, even when they may know them and how to communicate them.

The important aspect of any observation is the way in which they are marked. As far as possible, the rubric must focus on the quality of communication independently from the quality of the mathematics. It is possible to mark these questions from both points of view.

## Oral Communication

Oral communication obviously involves more than one person; thus, it is not surprising that we found any attempt to observe individual communication to be useless.

We found it difficult to observe oral communication if we focused on small groups, and also when we tried to observe it during lectures. In the first case, we did not feel that what we observed was a large enough communication sample to give a reliable result, even if we observed a pair or small group for a whole tutorial. The context of the tutorial and the composition of the group were so specific that no general comment could be made. In the case of lecture observations, there was rarely sufficient two-way communication to make any general observation.

The most successful observations of mathematical communication occurred in whole class observations during tutorials. We recommend the following optimal conditions:

- Students need to be working in groups of three (optimal), possibly pairs or fours.
- Students need to be working on complex or multi-stage problems, or open-ended situations—not on simple skills.
- Students need to be enculturated into working together—this often requires an initial “training” period where group discussion is promoted.
- The observer needs some experience or practise in making observations.
- The observer needs to be known to the students, and trusted by them.
- An observation schedule is useful, but cannot provide a complete picture.
- An experienced observer’s “gut feeling” appears to be reliable when backed up with observation schedule data.
- At least two full periods of observation are required to make any general statements.

Given these conditions, an observation of “mathematical communication in a class” can be obtained that will be sufficiently reliable to compare similar classes or to observe whether improvements have been made over the term of a course.

# Observing Written Mathematical Communication

## Setting the environment

If students are to have their mathematical communication observed, then they need to be informed about it. This means more than simply telling them that it will be observed, but includes telling them why it is being observed, including the value placed on mathematical communication as an undergraduate outcome. In addition, students should be told which questions are being marked for this characteristic. This will, of course, have the added advantage of increasing students' attention to this aspect of their studies.

It is likely that students may not understand what is meant by "mathematical communication". One way of highlighting this is to make public the marking rubric.

While we recommend using one or more communicative questions in every assignment, test and examination as a pedagogical tool, for the purposes of observing the development in communicative ability of a class, it is only necessary to have two such questions in the first half of the course, and two such questions in the second half of the course. Ideally the two questions will be in similar situations. For example, a question in the first assignment and the mid-semester test; followed by a question in the last assignment and the examination.

We were not attempting to detect changes in an individual's communicative ability in this way—and do not believe it is possible to do so. Evaluating an individual's written communication, would require more instances over a wider variety of situations. However, we did detect changes in the class's communicative proficiency with only two questions on two occasions. It is necessary that the questions are essentially the same type and length, the marking rubrics are the same, and the marker is the same (because marking such characteristics is more subjective than the usual type of marking that occurs in mathematics).

## Observation schedules

The schedules used in our trials were not closely related to those used in secondary mathematics in the literature. Secondary level mathematical communication seemed to be a different kind of skill to that which the lecturers were aiming at in undergraduate work.

We focused on having a rubric that was as simple as possible, so that marking large numbers of students' work could be done at least as quickly as marking standard mathematical questions. That is, a single viewing of the student paragraph, notation and graphs should be sufficient for a reliable mark.

After several trials, our marking schedules became designed around four aspects: the clarity, coherence and completeness of the communication (including focus); the accuracy of the communication; the use of more than one mode (prose, symbolism, graphs/diagrams); and "something extra", for example using more than one perspective, or relating the description to other mathematics.

We found that, with practise, we could quickly mark out of 10 using the schedule below:

| | |
|---|---|
| *Clarity, coherence, completeness, focus* | 1 2 3 |
| *Accuracy* | 1 2 3 |
| *Use of more than one mode* | 1 2 |
| *"Something extra"* | 1 2 |
| *TOTAL* | */10* |

No half marks were needed. We printed out slips of paper, circled the appropriate mark, and attached it to the script.

## Observation Results

It needs to be reiterated that, as an individual measure, such rough marking is not particularly reliable. It is very question dependent.

Nevertheless, we found that the very process of having an assessment of communicative ability encouraged students to focus on this task. It was not surprising, therefore, that later assessments tended to have higher marks.

The distribution of marks over a class of more than about 40 students was stable (we did not undertake statistical testing for reliability). Our experience was that the class results were therefore useful for comparing classes or observing improvements (or otherwise) over the course of a semester.

## Things to watch out for

We found it very difficult to predict which questions would be effective. Even after some practise, a question which we thought would work well turned out to be a complete failure—usually because the communicative task was interpreted very differently by different students. We recommend therefore, that questions be trialled if possible, that questions specify clearly what is required, and that lecturers be prepared to abandon a question if it proves to elicit too many different types of response from students.

Our trialling of written communication questions showed us that students were less able to perform this task than we expected. It highlighted the need both to emphasise this skill to students explicitly, and to model good communication. We recommend that, early in an undergraduate student's career, they are exposed to model answers for this type of question so that they have some idea what is required.

# Observing Oral Mathematical Communication

## Setting the environment

Setting up an environment where a significant amount of oral mathematical communication occurs for all students is not easy. Some students are natural communicators, seeking out peers and tutors, and having the required social skills to initiate or request discussions, and keep them going. Other students actively avoid mathematical discussion, preferring to work individually and using texts or web resources.

The bullet points on page 5 detail what our project found to be the optimal conditions for establishing simultaneous communication for most students in a tutorial. Key conditions are: groups of three; appropriate tasks; some initial "training"; and awareness that communication is being observed.

We recommend that groups of three be established from the very first tutorial meeting of the students, and strictly enforced. We preferred groups of two, rather than a group of four, if the number of students was not divisible by three. It does not seem to matter significantly whether the group composition is decided by the students or the tutor.

The initial training can be quite light. It will likely be sufficient for appropriate behaviour to be described (discuss what you need to do, work together where possible, discuss the answer(s) or results) and then set a task, and go around the groups focusing on this behaviour, ensuring isolates become part of the group, and praising good communication. It may be necessary to implement other strategies. These include:

- Requiring all work to be done on one common piece of paper or whiteboard.
- Sitting in with a group and modelling good communication, and being inclusive of others.
- Asking the group to prepare a joint presentation to the rest of the class.

In the second tutorial, a reminder is probably all that is required.

When an observation is to occur, students should be informed that, in this tutorial, the whole class is being observed for their communicative behaviour. The observer will ideally be known to the class (e.g. a regular tutor), but will not act as a tutor on that occasion.

We recommend that the observer be given the opportunity to practise observing a class during a tutorial. This could be done for a group of tutors by simply asking them to notice the levels of communication, and then have a short group discussion sharing what was seen and becoming familiar with the observation schedule.

## Observation schedules

The observation schedule is best taken as a supplement to a "gut feeling" observation. The "gut feeling" observation can be a single mark out of five given at the end of the session. Our experience was that observers being asked to make such an evaluation will quite quickly find their own standards and become sufficiently reliable. More about "gut feeling" judgements in a teaching environment is available from Ell and Haigh (2015).

We recommend that the observer chooses three groups and observes them for three minutes each on three occasions (beginning, middle and end of the session). Immediately after each group observation the schedule should be completed. In any spare time, the observer should stand back and watch the whole class.

While watching the whole class, the following questions are important:

- Are groups in general all participating, or is one person dominating?
- Are the groups making progress on the problems, or going around in circles or getting distracted?

- Are the groups using diagrams, gestures, technology to support their communication?
- Is mathematical terminology and jargon being used?
- What happens if a member of the group does not understand?
- Is their communication organised and explicitly managed?

While observing each group, the observation schedule in Appendix 2 can be used. One schedule is needed for each group, but all three observations of that group are made on the single sheet. There are three observations to note.

1. A single overall assessment of the group interaction during the three minutes.
   a. **X** if the interaction is predominantly off-task or absent.
   b. **M** if there is some, but minimal interaction on-task (may be organisational only).
   c. **S** if there is significant communication (discussion, questions, explanations) on the mathematical topic.
   d. **H** if the communication is high level, e.g. involving synthesis, justifications, and challenges to each other.
2. A single assessment of the role played by each individual. A number will need to be allocated to each member of the group. This could be done, for example, by numbering as 1 the person with their back to the door, and proceeding clockwise.
   a. **X** If the person makes no communication and is not listening.
   b. **L** if the person makes no contribution, but is listening to the others.
   c. **C** if the person contributes in some way (mathematically or organisationally).
   d. **CX** if the person makes a contribution but dominates and blocks others' contributions.
   e. **S** if the person makes a consistent, multifaceted contribution, including at least three of listening, responding, initiating, questioning, justifying or organising.
3. A note of any aspect of the three-minute observation that was significant and not accounted for in 1 or 2 above. (Typically this might occur two or three times over the course of all 9 group observations, i.e. mostly this part will be left blank).

We recommend using a clip-board. It will be necessary for a stop-watch (on a cell-phone?) to be used so that the three minutes is adhered to.

## Observation Results

The nine observations should be taken together, and an overall mark out of five given for the class. This mark can be added to the "gut feeling" observation to give a mark out of ten.

We believe that this mark out of ten is robust enough, and accurate enough, to compare classes and to observe changes in the communicative behaviour of a class over the course of a semester. We do not think a more accurate observation is possible without considerably more effort.

## Things to watch out for

The subjectivity of the observations is obvious. However, observers will quickly become internally consistent. Therefore, provided the observer is the same, reliable judgements can be made. Two observers will become externally consistent after one semester of experience, combined with regular discussion of their observations.

The very act of observing oral communication in tutorials has an effect on the students. Therefore, an improvement over the course of a semester is to be expected. If there was no improvement, then that would be a matter for concern.

Observing is a non-trivial task. Observers need to concentrate hard for the full tutorial, and are not in a position to take up normal tutoring duties. We recommend that, when an observation is taking place, other tutors are available so the observers are not distracted.

# Appendix 1

## Sample examination questions for testing written communication

### 1. Suitable for an entry level Year 1 calculus course

You are revising for the mid-term test with a friend who has missed a number of lectures.

You tell your friend that $f(x) = x^2 + 3x + 1$ has derivative $f(x) = 2x + 3$

Your friend does not understand what this means. How would you explain this concept to them? Your answer should be less than half a page.

Your explanation must include words, symbols and diagrams.

*Alternative formulation:*

You are revising for the mid-term test with a friend who has missed a lot of lectures.

You are doing a question in which you need to find the co-ordinates of the turning point of $f(x) = (x - 3)^2 + 4$

Explain to your friend how you know that the turning point will be at (3,4).

A good explanation will include words, symbols and diagrams.

*Or:*

You are revising for the examination with a friend who has missed a lot of lectures.

You are doing a question in which you need to find the co-ordinates of the point on the curve $f(x) = 3(x + 2)^2 - 5$ for which the function has a minimum value.

Explain to your friend how you know that the minimum value will be $f(x) = -5$ when $x = -2$

NOTE: In our trial the first option above was the most difficult to mark because it was too broad. Students understood the phrase “explain this concept” in different ways, some considering differentiation as a whole, some focusing on process, some on the idea of a gradient function.

### 2. Suitable for a Year 2 linear algebra course

When calculating the inverse of a matrix, part of the procedure is to divide by the determinant of the matrix. A friend asks you to explain why it is necessary to do this: what is the purpose of this operation?

Write a paragraph answering this question. Include a comment about the matrix that would be obtained if division by the determinant was NOT undertaken.

# Appendix 2

## Group and Individual Oral Communication Observation Schedule

| Group: | Date: |
|---|---|

| Type of Observation | Start Session Observation | Mid Session Observation | End Session Observation |
|---|---|---|---|
| *Group Interaction:*<br>**X** — Off task<br>**M** — Minimal<br>**S** — Significant<br>**H** — High level | | | |
| *Individual Interaction:*<br>**X** — No participation<br>**L** — Listening only<br>**C** — Contributes<br>**CX** — Blocks others<br>**S** — Superior | *1:* *2:*<br>*3:* *4:* | *1:* *2:*<br>*3:* *4:* | *1:* *2:*<br>*3:* *4:* |
| *Other items of note* | | | |

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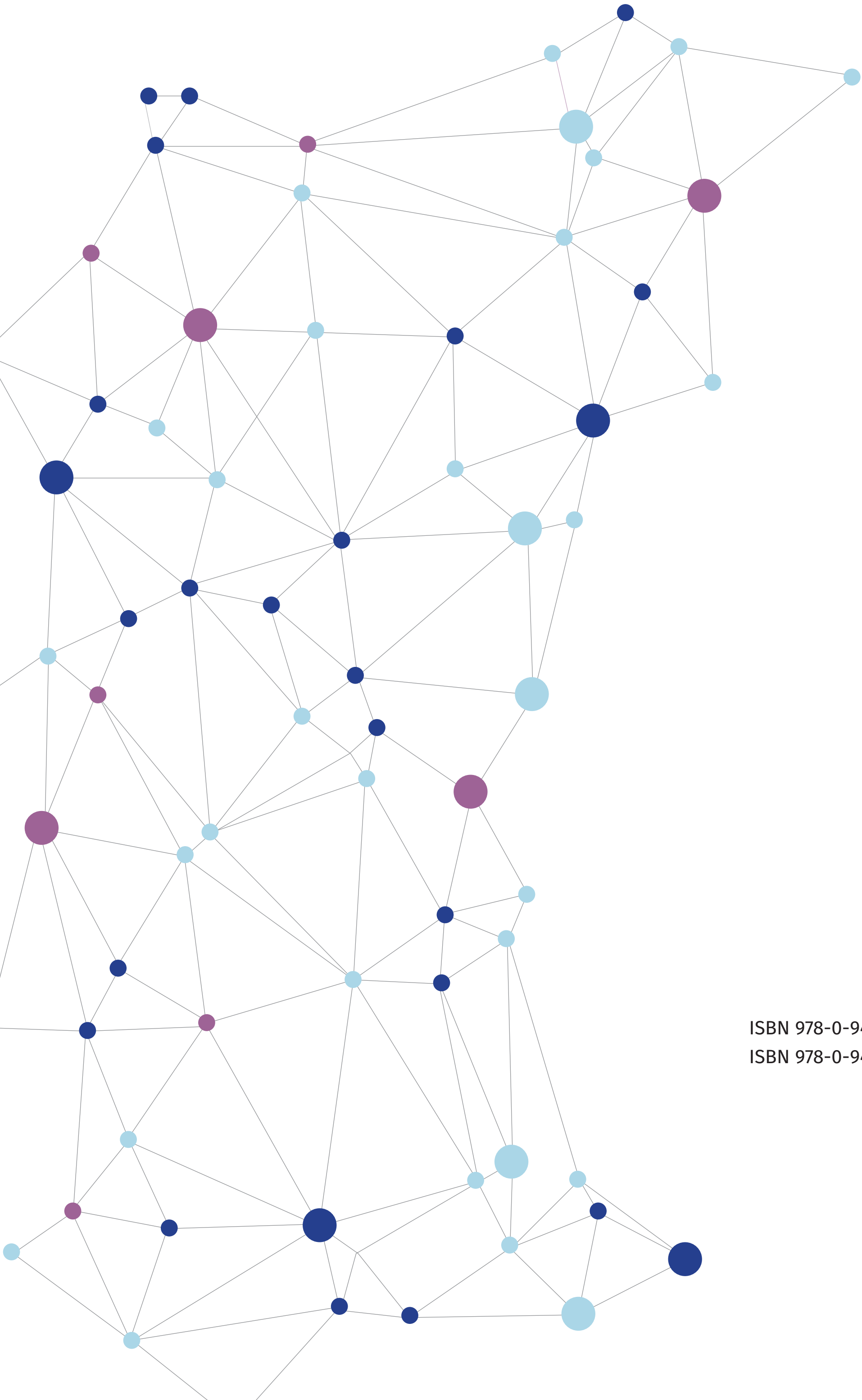

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