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Mathematical Reasoning and Knowledge in Initial Teacher Education (MARKITE)

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Introduction

The New Zealand Government's goal is to have an education system that equips all learners with the knowledge, skills, and values needed to be successful citizens in the 21st century (New Zealand Government, 2011); government policy recognises that mathematical understanding is fundamental to effective participation in society, and delivers significant social and economic benefits (Tertiary Education Commission, 2012). Unfortunately, studies suggest that a significant proportion of the adult population does not have the level of mathematical thinking needed to function effectively in society (Alkema & Rean, 2014; Department of Education and Skills, 2011; Satherley, Lawes, & Sok, 2008). Mathematical thinking and reasoning skills are embedded in many of the activities that are integral to active participation in society, although we do not always realise this is the case (Atweh & Brady, 2009; Barwell & Suurtamm, 2011). For example, most sports rely on some understanding of mathematical and/or statistical ideas, and household budgeting and time management certainly do. Mathematical thinking is increasingly implicated in many workplace activities and careers (Skills Organisation Incorporated, 2014) and in many of the political and environmental issues that challenge society today (Geiger, Forgasz, & Goos, 2015; Steen, 2001).

Teachers, with or without direct awareness, use mathematical thinking in many aspects of their professional work, as well as in their personal lives. They use mathematical thinking in class and school administration, to analyse quantitative and qualitative student data as part of evidence-informed teaching and learning, and they teach and use mathematical thinking in their delivery of the curriculum as a whole (Goos, Geiger, & Dole, 2012; Leder, Forgasz, Kalkhoven, & Geiger, 2015; Mandinach & Gummer, 2013). Advocacy for formative assessment/assessment for learning (and data-driven/evidence-based decision making) in government policy, and initiatives that require teachers and schools to account for student learning are lifting teacher mathematical thinking demands (Ministry of Education, 2011; Wayman & Jimerson, 2014). Thus, while teaching the mathematics curriculum is part of what some teachers do, all teachers engage with mathematical thinking in the course of their work.

A high proportion of primary student teachers enter teacher education with a limited range of mathematical experiences and show little evidence of connected thinking that recognises the place and role of mathematics (Chen & Mu, 2010; Witt, Goode, & Ibbett, 2013). There are concerns worldwide about teacher content knowledge in mathematics (Lowrie & Jorgensen, 2016; Schmidt et al., 2007; Young-Loveridge, Bicknell, & Mills, 2012). This has led to some jurisdictions raising the mathematics qualification entry and/or exit requirements for Initial Teacher Education (ITE) programmes (Means, Chen, DeBarger, & Padilla, 2011). Since 2013, prospective teachers in the UK have been required to pass a professional skills assessment in mathematics before gaining entry to an ITE programme. In New Zealand, the Education Council has required higher level mathematics assessment for new Master's-level ITE programmes, and a recent position paper *Strategic Options for Developing Future Oriented Initial Teacher Education* (Education Council New Zealand, 2016) suggests that the mathematics requirement for ITE, which currently specifies University Entrance Numeracy, needs to be raised. Internationally, there is an ever-expanding body of research and associated professional programmes focused on developing teacher mathematical content knowledge and data literacy, all of which suggests attention to teachers' capacity for mathematical thinking is a matter of strategic interest worldwide.

The context for the study

The New Zealand Curriculum (Ministry of Education, 2007), *Te Mauratanga o Aotearoa* (Ministry of Education, 2008), and *Te Whāriki* (Ministry of Education, 1996) all have elements of mathematical thinking embedded across the breadth of their learning goals (Cowie & Cooper, 2016). This is important to note because research has shown that the capacity to deeply understand, identify, and use mathematical thinking across a range of contexts requires experience of its use beyond the mathematics classroom (ACER, 2009; Human Capital Working Group, Council of Australian Governments, 2008; Steen, 2001). The *New Zealand Graduating Teacher*

Standards (New Zealand Teachers Council, 2007) require graduating teachers to be able to demonstrate numeracy competencies relevant to their professional role. The introduction of National Standards (Ministry of Education, 2009) in mathematics in New Zealand primary schools has reinforced the importance of student mathematical thinking and, concomitantly, the expectations that teachers are able to use assessment data to develop the mathematical thinking of diverse students (Chamberlain & Caygill, 2012).

For the purposes of this study, we take the mathematical thinking and reasoning teachers need for their professional role as that required for:

i. teaching the mathematics embedded across all curriculum learning areas
ii. collecting, interpreting, reporting, and acting on student achievement data
iii. carrying out administrative tasks.

The Mathematical Reasoning and Knowledge in Initial Teacher Education [MARKITE] study sought to achieve the development of the above aspects of mathematical thinking through a focus on supporting student teacher mathematical thinking across the breadth of their ITE programme. Specifically, the research sought to develop student teachers' competence in, confidence with, and critical awareness of mathematical thinking through a combination of self-regulated and embedded learning experiences. Self-regulation, adaptive help seeking, and participation in learning communities are important skills for teachers if they are to continue to learn and to equip all learners with the knowledge, skills, and values needed to be successful citizens in the 21st century (Hattie & Donoghue, 2016; Steed & Poskitt, 2010).

Focusing on a 1-year primary graduate ITE programme, the study investigated the benefits of embedding, and making explicit, mathematical thinking and reasoning within the courses across the programme alongside the provision of mentoring and website support, on student teacher competence, confidence, and critical awareness of mathematics. Aiming to support independent learning into the future, the project had two levels of focus: individual student teacher learning and the system that supports this learning. The research questions for the study were:

1. Which resources and practices are effective in supporting the development of student teacher mathematical thinking within a tertiary setting?
2. How does student teacher mathematical thinking change over the course of a 1-year graduate diploma ITE programme when student teachers are provided with embedded and self-regulated opportunities to learn?

We begin the report by describing the research design and the data sources for the project. In the findings section of this report we firstly describe the school context for the use of mathematical thinking and reasoning to confirm the conceptual framing of the project. Secondly, we outline the findings related to the effectiveness of resources and practices put in place to support student teacher mathematical thinking development (Research Question 1), and thirdly, we describe findings on the nature of and changes in student teacher mathematical thinking and reasoning, and awareness over time (Research Question 2). We conclude by discussing the implications of the research and its limitations.

The research design

The 3-year project used a combination of a design research approach (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) and a design-based intervention research approach (Penuel & Fishman, 2012) to investigate the research questions. That is, we aimed to engineer change but paid equivalent attention to theory development. Data sources to understand this process were as follows.

Research Question 1

Contextual information

To confirm the conceptual underpinning of the project, and to assist us to answer Research Question 1, stakeholders from schools were interviewed. Data collected included:

i. *Focus groups with principals, beginning teacher coordinators, and data managers*

   These were held in year 1 and year 3 of the project. Data helped us scope the mathematical thinking and reasoning proficiency valued by schools.

ii. *Focus groups with year 1 and year 2 beginning teachers*

   In 2015, six year 1 beginning teachers who had completed their ITE programme in 2014 were interviewed. In 2016, two focus groups were held with 10 beginning teachers across four schools. These groups represented teachers who graduated from both graduate and undergraduate programmes and who were in their first and second year of teaching. Using a mediated conversation data collection technique (Cowie & Hipkins, 2014) they were asked to share artefacts that demonstrated how they used mathematics in a curriculum area outside of mathematics, the type of information they collected about student achievement, and an example/description of how they used mathematics in administration.

Resources and practices

To answer Research Question 1 we aimed to identify system-wide processes that supported a programme-wide approach to embedding mathematical thinking. The system-wide approaches included strategies for students and lecturers. We identified and mapped the mathematical thinking evident in course outlines and provided teachers with the maps. We interviewed lecturers about their perceptions of the place and relevance of mathematical thinking in their courses and asked lecturers to highlight to student teachers what mathematical thinking and reasoning was embedded in their courses. Students had access to opportunities to talk to a mathematical mentor and to a website dedicated to making visible mathematical thinking in different contexts. In this way, we anticipated student teachers would experience mathematical thinking in action across a range of contexts, which is thought to support deep understanding (Nuthall, 2007).

Data collected included:

i. student teacher surveys and assessment, individual and group focus group interviews
ii. interviews and focus groups with Graduate Diploma of Teaching Primary programme course lecturers over the 3 years, prior and post the teaching of their course
iii. observations of lecturer classroom practice
iv. analysis of course documentation, in conjunction with interview data, to produce curriculum maps showing opportunities for mathematical thinking at the beginning of each semester and post teaching their course to ascertain their experiences of doing.

Research Question 2

Nature and changes in student teacher thinking

To answer Research Question 2 we interviewed and surveyed student teachers to identify areas of mathematical thinking and reasoning where they required support with confidence, competence, and critical awareness and provided self-regulated resources through mentoring and a purpose-built website. We tracked changes in their learning over the course of the year. Data collection included:

i. student teacher surveys at the beginning and end of their year-long programme each year for the 3 years of the study
ii. student individual and group interviews at the beginning of their programme and pre and post practicum discussions each year for the 3 years of the study
iii. mentor self-reflections at the end of each year about student teacher learning and engagement.

Key findings

(i) The school context for mathematical thinking (Research Question 1)

Principals' perspectives

At the beginning of the project a focus group with principals to inform the development of self-regulated resources and the website confirmed that mathematical thinking and reasoning had an important part to play in teachers' work. The principals did not have any substantive expectations of beginning teachers mathematically, apart from knowing the structure of the mathematics and statistics learning area. They emphasised the value of a positive attitude towards mathematics and learning mathematics if and when required, and the ability to recognise opportunities to include mathematics in a crowded curriculum. The ability to work collaboratively with other teachers and knowledge of assessment processes such as the role of the self-assessment in children's learning were also highlighted. The principals noted that beginning teachers would be expected to assist with but not lead administrative tasks associated with activities such as overseeing and preparing budgets, timetables, organising sporting events and school trips. They acknowledged that the ability to integrate ideas and make links comes with experience and considered that induction and mentoring programmes were crucial in supporting beginning teachers with their mathematics programme, with the development of data literacy and assessment skills, and for administrative tasks. In the same conversation, principals pointed out that many experienced teachers are not confident teaching mathematics and needed support with data analysis and interpretation. They reported that they often made assumptions about teacher knowledge and also that they provided tiers and layers of support for the teachers for whom they were responsible. They suggested a range of appropriate resources that would be appropriate for student teachers.

In the third year of the study, the interviews with school data managers identified the importance of a critical stance, positive disposition towards learning, and the ability to work in teams as essential attributes to contribute to teaching programmes and to develop data literacy within classrooms and schools. Data managers commented they wanted beginning teachers who had some proficiency in the collection, interpretation, and action on data but emphasised that they expected to work with their beginning teachers collaboratively to develop these capacities. School mentors were expected to support beginning teachers to moderate and understand their data in relation to wider school data at National Standard and/or curriculum level.

Beginning teacher perspectives

The beginning teacher interviews provided evidence of how their learning and the resources they had been introduced to in their ITE course had assisted them in their first year of teaching. They were confidently able to describe how mathematics was embedded across the curriculum and their practice to support this. Artefacts shared included a financial literacy task that required children to create and run their own businesses, which in turn required them to form budgets, establish market prices, and calculate profit/loss. One teacher shared how she had integrated mathematics into a Te reo Māori unit including geometry (Shapes—Ngā Āhua; te whānau Taparau—Polygon family) and a lesson where students created a range of polygon shapes and learned their Māori names. Use of student achievement data included a range of tools including GLOSS, IKAN, PATs, e-asTTle, and running records. These were used to form groups and to target one-to-one time with students.

The beginning teachers shared examples of templates that they used to record progress against National Standards and templates their students used to track their own achievements. Numerous examples were shared of mathematical thinking used for administration such as budgeting, organisation of class activities, and so on. They identified skills they needed as beginning teachers such as statistical knowledge (creating tables, using Excel spreadsheets) and basic addition and subtraction and multiplication and division knowledge and strategies in order to be able to group, allocate budgets, etc. The beginning teachers could identify parts of their ITE coursework that had supported them to do these tasks and made suggestions for improvement.

(ii) Lecturer support for using embedding and developing mathematical thinking and reasoning (Research Question 1)

Initial meetings

The initial meetings with lecturers were very important to clarify with lecturers the researchers' views of mathematical thinking and its role in teachers' work. The discussions helped develop a shared understanding of the project; that is: (i) the project was not about mathematics teaching but the mathematical thinking used to support ideas in non-mathematics courses, and (ii) mathematical thinking was not just about numbers. The meetings also assisted in consolidating the researchers' understandings.

Through these discussions, lecturers endorsed the project and came to appreciate the extent to which mathematical thinking might be an aspect of their course curriculum, albeit often as hidden or implicit aspects. They were very clear they did not see it as their role to teach mathematical or statistical thinking outside the context of its use in their courses but they did come to realise how it might be embedded and made more explicit. They found cross-programme discussions and the curriculum maps developed within the project valuable in helping them seeing and making connections across the programme and to come to a shared understanding of what was meant by mathematical thinking.

Ongoing lecturer interviews

In the interviews held in the first year, most lecturers were initially dubious that they would have anything to contribute to a discussion about mathematical thinking and some expressed their dislike of mathematics, asserting that they had not enjoyed it at school. Nonetheless, all lecturers, with prompting, were able to identify and describe aspects of mathematical thinking relevant to their course. These interviews, held five times over the course of the project, typically at the beginning and end of a teaching semester, were important in embedding the project as an integral part of the programme and supported the development of lecturer understanding of the mathematical thinking as distinct from mathematics teaching. The lecturers willingly participated and shared their evolving practices including strategies and artefacts. As the project progressed, they reported that they actively incorporated and highlighted mathematical ideas during their teaching.

Mathematical thinking as a tool for critically analysing data was often embedded in course concepts although lecturers indicated they did not always make this overt to students. It was also established that some lecturers were making assumptions about student teacher skills for activities such as interpreting data and graphical information. Through these conversations, we identified a further context—mathematics for political action. A number of lecturers touched on the political use and social meaning of numbers in the current context of accountability and competition; for example, in PISA country rankings which were a focus of media and policy attention around the time of the interviews. One education lecturer described in depth the need to help students to interpret and then understand the social and political implications of the numerical categorisation of children with different disabilities (Olsen, 2015). Lecturers indicated that they found reflecting on what mathematical ideas might be embedded in their paper and cross-paper conversations valuable and for some it had changed their conception of mathematical thinking. As one commented:

> My traditional view of maths thinking was of 2 + 2 = 4. I wasn't thinking of maths conceptually. I wasn't thinking of it as philosophical tool to theorise [people] through number. I now realise I use numbers to interpret the nature of the relationship between stats and policy. *(Education lecturer, reflective interview, 2016)*

Curriculum mapping

Preparation of curriculum maps from document analysis supplemented by interview data and feedback from lecturers also assisted in deprivatising what was included in coursework. Making this transparent facilitated a better overview of the programme for all participants.

Analysis of course outlines provided us with an initial programme curriculum map. We mapped identifiable mathematical thinking against the three contexts of interest in the study: teaching across the curriculum, data literacy, and administration. We also noted through the document analysis that course assessment tasks and course information often involved mathematical thinking. The map was further developed using lecturer and student teacher commentary. While this information (often more specific and detailed) enhanced the first map, it was clear that the researchers and the lecturers needed greater clarity and specificity about what mathematical thinking is, and to reach a shared understanding, so that the mathematical thinking could be more easily recognised and, in due course, highlighted to students. Revisiting the literature, we identified five categories of mathematical concepts to help frame the further development of the map: Pattern and relationship finding; Using number to solve problems; Reasoning statistically; Dealing with uncertainty; and Constructing and critiquing representations. We developed a template that specifically asked for examples of each of the five categories, and provided prompts in the form of two or three authentic examples already identified by lecturers. This process over two iterations of map development showed us that (i) more clearly identifying the mathematical thinking and (ii) continued dialogue between researchers and lecturers were both helpful in creating a more useful map.

The process of developing the map—the course document analysis, interviews, the use of the mapping template, and the map itself played a valuable role in expanding and clarifying what mathematical thinking is and in identifying opportunities for learning related to mathematical thinking in the ITE programme. The conversations between lecturers and researchers served to stimulate thinking amongst lecturers about how they could highlight for students the mathematical thinking that was embedded in their courses and where it might be appropriate to focus more strongly on it. In the second year of the study the Social Studies lecturer offered a new assignment in which pre-service teachers could choose to investigate the topic 'Numeracy in Social Studies'. The Professional Practice and Inquiry 2 course required students to develop a resource for parents on formative assessment which could involve the mathematical idea of statistical reasoning and the Professional Practice lecturer focusing on inclusion paid more explicit attention to student understanding of data tables.

Overall, comparative analysis of 2014–16 data showed a growing awareness of mathematical thinking by lecturers in six of the 11 non-mathematics courses. Five of the six lecturers interviewed in semester A of 2016 described changes to their papers and impacts on their practices and teaching:

> I was thinking, you know, when I first sat down with you guys, I was quite sceptical, 'cause the area that we work in is very much dominated by the results of what happens when you're given a number. I've been a lot happier, about including numbers, and making it work for the kinds of inclusive ideas that we're using. And I think that was a challenge from the project, and for my initial response—I had to go back and think again of the value of using mathematical concepts as a tool to effect change. I didn't think that I had anything to contribute to this project. But in fact, it really did make me think about what is the role of number in a different way. And it's made me a lot more comfortable with the mathematical implications of my field, definitely, which are huge. So that—if that's an outcome... Well I think it's a good outcome. (*Education lecturer, reflective interview, 2016*)

Observations of classes in 2016 confirmed that some lecturers were paying explicit attention to mathematical concepts with lecturers highlighting the mathematical thinking involved. For example, a Social Studies lecturer guided students through interpreting tabular data; a Professional Practice and Inquiry lecturer worked with students on PAT data, bar graphs, and box graph plots; and a Science lecturer showed students how to work with variables and create tables from observational data as part of an in-class experiment.

The following comments, made by a Science lecturer and a Social Studies lecturer in an interview in 2016 are representative of the perspectives of the lecturers on the impact of the research in which the curriculum mapping process played a central role:

> I think the things that have changed in the last two years, having started thinking about this, is that the maths, rather than just assuming, and rather than just doing it, you make it a lot more explicit ... I think it's just more the teaching approach that I take—it is what I am thinking about more. When they sort of make their little whirly birds or helicopters. You know, they're measuring the length, so someone will say, "Oh, I've made the wings longer." And I'll say, "So how much longer did you make them?" So, make them come back to a number. So 'longer' and 'shorter' is not necessarily a good way of describing, you know, the factors that you're trying to

> change or measure. I'm wanting you to give a specific numerical amount. I think it's about the care and the way in which I'm doing things is better. (*Science lecturer, reflective interview, 2016*)

> Well we've looked at ways in which we can get Maths into the paper. One of the things that I think is important is that there's the misconception in schools that numeracy and literacy are separate from the other subjects. And so that's the point we've been making for students, is that Social Studies is absolutely reliant on literacy and numeracy. It's not a subject that stands in isolation of those two things. And so over the last two years, particularly in terms of readings, I've looked for more that actually show the possible things you can do related to numeracy in the context of Social Studies. We were doing virtually nothing two years ago. (*Social Science lecturer, reflective interview, 2016)*

Curriculum mapping is increasingly being used to understand cross-curricula ideas and to identify gaps in a curriculum (Kertesz, 2015). Artefacts that act as boundary objects (Akkerman & Bakker, 2011), such as our mapping template, are known to be useful in a curriculum mapping process as they facilitate communication and cooperation across different groups and communities. This coordinated approach also expanded lecturers' definition of what mathematical thinking was and led to some changes in their practice.

(iii) Development of student teacher awareness of mathematical thinking (Research Question 2)

The approach to this question involved making students aware of gaps in their knowledge, alerting them to the mathematical thinking embedded in their coursework through teaching and curriculum maps, and providing mentoring support and a website containing self-regulated resources—all strategies identified as appropriate for adult learners (Fletcher, 2007; Harlow, 2013; Ross, 2011). Zepke and Leach (2010a, 2010b) and Zepke et al. (2010) observed that boosting students' success requires teachers and institutions not only to improve the quality of teaching, but also to encourage the development of collaborative learning communities, and to support learning outside the classroom.

Changes in student teacher mathematical thinking and awareness over time

Findings on the nature of and changes in student teacher perspectives relate to the following aspects of student teacher knowledge and views: (a) mathematical content knowledge and confidence; (b) attitudes towards and views of mathematics; (c) use of self-regulated resources; and (d) changes in awareness of the existence of mathematical thinking.

a. Mathematical content knowledge and confidence

Student teachers' mathematical content knowledge and confidence (number)

Student teachers' mathematical thinking competence and confidence was gauged in order to understand where they might need support. The tools used to do this evolved over the 3 years as the team sought to find an instrument that would generate information of a nature and depth that was useful, did not add to student teachers' anxiety about mathematics, and was also time efficient. Student teacher content knowledge, mainly number knowledge, and data literacy were assessed as well as their confidence and attitudes and awareness.

Initially we surveyed student teacher mathematical content knowledge using items from the UK Qualified Teacher Status (QTS) numeracy skills test (Department for Education (UK), 2012). These were set in real-life contexts; a majority of students were able to answer the questions. In the second and third years, the research team sought and gained consent to use the number content knowledge test developed and used by Jenny Young-Loveridge and colleagues (Young-Loveridge et al., 2012). These questions had the advantage that they were locally developed and validated. In both years, around three-quarters (76% and 77% respectively) of the student teachers responded correctly to more than 50% of the questions. In all 3 years, students had most problems with questions dealing with fractions (i.e., equivalent fractions), percentages, and decimal places.

Consistent with the project emphasis on confidence, student teachers were also asked to rate their confidence that their answer was correct for five of the 10 number questions in 2015 and 2016. In both years over half of students across all questions felt confident to a certain degree of their answer. Students felt the least confident when the problems dealt with proportions. Interestingly even when problems were 'simple multiplication', between 30% and 40% of students did not feel fully confident that their response was correct even when a majority gave a correct answer. On the other hand, some students felt either somewhat or fully confident when in fact they provided an incorrect answer. Student teachers being confident when their answers were not correct and feeling not confident when their answers were correct are both equally problematic because of the potential for misguided action/in-action (Foster, 2016). However, in considering these results it is pertinent to note that focus group students in 2016 commented that they had 'forgotten' and/or 'lost touch' with the kinds of mathematical thinking in the questions because "we're not using them in our everyday life". Given the focus on number operations, these comments suggest that to these students some of the kinds of mathematical thinking that they were likely encountering in their daily lives were invisible to them. This is consistent with Greer and Skovsmose's (2012) work on critical numeracy which purports that mathematics is often kept hidden within social and political contexts and in decision making.

Student teacher mathematical content knowledge and confidence (statistics/data literacy)

Initially, the content survey included five questions that required students to extract information from graphs and tables and to calculate percentages. When students had to only read the information from graphs, most (79% to 92%) students gave a correct answer. When students had to use/calculate proportions in addition to reading a graph, around two-thirds (60%) provided a correct answer. Responding to these results, in the next year we replaced these questions with 14 data literacy questions sourced from the UK QTS numeracy skills test (Department for Education, UK, 2012). The questions were reworded and made appropriate for New Zealand context. On average, students provided 11 correct answers. Students had most difficulty with drawing graphs free-hand. Again, our analysis indicated that the questions did not provide sufficient differentiation for us to be able to confidently identify student learning needs. In 2016 we sourced and used a set of questions from the Tertiary Education Commission document *Teaching Adults to Reason Statistically: Using the Learning Progressions* (Tertiary Education Commission, 2015). The first five questions dealt with reading graphs and the following six focused on statistical/data reasoning and making inferences in context of glasses of people drank water per day. For each question, students were asked to rate their confidence in their answer. At the end of the year we changed the context to class marks. The quality of students' statistical reasoning did not change substantially by the end of the year. Of note in relation to this task, student discussions amongst themselves and with the research team post completing the data literacy questions indicated the questions and the assessment experience doubled as a powerful learning experience with conversations involving attempts to recollect and offers to explain ideas. At the end of the year, where the context for the same questions was switched to student assessment data, the conversations revolved around clarifying definitions and seeking/detailing occasions where the ideas had been used during practicum. In practice, this task proved to be a powerful teaching and learning activity.

In terms of confidence, as a general pattern, a majority of students (over 85%) felt fully or somewhat confident when reading a simple graph but less confident with free-hand graphing (around 50%). Students' level of confidence dropped from predominantly fully confident to predominantly somewhat confident or not confident in the domain of statistical reasoning (predicting and inferring progression). The drop in student confidence evaluations paralleled the drop in the accuracy of their answers. As in the 'numbers' part of the assessment, some students felt fully confident that they were correct even when they gave an incorrect answer, and vice versa.

b. Student attitudes toward and view of the nature of mathematics

Students' attitudes toward mathematics

Students' attitudes toward mathematics were measured using the Attitudes Toward Mathematics Instrument (ATMI) (Tapia & Marsh, 2004). The ATMI consists of 40 items representing four subscales: self-confidence/sense of security (16 statements); value, importance of mathematics (10 statements); motivation to learn mathematics (7 statements); and enjoyment of mathematics (7 statements). Students responded to the statements using a 5-point Likert scale from 1 (strongly disagree) to 5 (strongly agree). Negatively worded items were reverse scored. The scores on items were summed to create four subscale scores. Higher scores indicate higher levels/ more positive attitudes toward mathematics. The reliability estimates for each of the four subscales were adequate and were comparable to findings by Sundre, Barry, Gynnild, and Ostgard (2012).

On each scale, the students' group average score was higher than the half-range value for that scale. These results are consistent with those observed in the literature using the Tapia and Marsh instrument. There were no statistically significant differences between students' average scores across three cohorts (2014, 2015, 2016) on any of the scales. Interestingly, this pattern was consistent in other groups (undergraduate programmes and Master of Teaching and Learning programmes) given the same survey.

Students' views of the nature of mathematics

In all 3 years the same eight items were used to assess students' beliefs about mathematics learning. They were selected by the mathematics educators on the team to reflect key themes in the literature. The students responded on a 5-point Likert scale from 1 (strongly disagree) to 5 (strongly agree). Across the 3 years around three-quarters of students agreed that some people have a maths mind and that when two people do not agree they need to ask the teacher. In 2014 and 2016, nearly two-thirds of students agreed that it is important to get the answer right, while in 2015 only a third of the students agreed. These beliefs about mathematics and mathematics learning could be referred to as 'traditional' and to position teachers as having the responsibility for transmitting knowledge that needs to be learned. On the other hand, in all 3 years, a majority of students thought it was important for students to be able to explain how they had solved a problem and that knowing why an answer is correct is just as important as getting the right answer. These student beliefs resemble, to a certain degree, an inquiry-oriented approach to learning mathematics and a conception of mathematics as a tool for problem solving. This general pattern of responses was maintained across the three cohorts (2014, 2015, 2016). Too few students responded to this survey at the end of the year to be able to make a judgement about any change in their understanding of mathematics but their initial views suggest they held views that were in tension with each other.

Overall, the mathematical thinking survey results suggest the students in our study struggle with many of the same aspects of mathematical and statistical thinking identified in other studies and have a similar profile to those of practising primary teachers in New Zealand (Ward & Thomas, 2007). While most students could see the value of mathematics, a significant number reported they lacked confidence and motivation and did not enjoy mathematics. This student teacher response is cause for concern given the shifting, and increasing, demands teachers face to make sense of quantitative student achievement data. The results raise three points: (i) they come to ITE with variable understandings; (ii) they have variable capacity to self-assess the nature of their understanding as evidenced by the relationship between their confidence in their answer and its actual correctness; and (iii) it seems that the mathematical thinking they have been using in their everyday lives may have been invisible to them or simply not used. Each of these points and their possible interactions pose a challenge to ITE programmes in relation to how they might best support and progress student teacher learning.

c. Role of self-regulated resources and mentoring (Research Question 1)

Role of the assessment

The assessment process on the surface did not appear to alert students to the necessity to access mentoring or self-regulated resources. There was very little discrimination across the cohort in the content assessment but it alerted us to a lack of proficiency in proportional reasoning and aspects of data literacy. Very few students approached the mentors to discuss their assessment and to seek support.

Role of mentoring

Despite using a variety of ways to communicate with student teachers to alert them to opportunities to work with mathematical thinking mentors, including group and individual email invitations, posting information about their role on Moodle, general invitations during course lectures, and the offer of dedicated data literacy workshops (these were run by another member of the team), very few students took up these opportunities.

One thing to note is that the mathematics education course completed in the first half of the semester had a significant focus on conceptual understanding of proportional reasoning. Students commented that they felt well supported in this paper. This teaching happened after the invitation to participate in mentoring.

In response to the mentoring participation rates, we implemented pre and post practicum discussions for student teachers to discuss what they might see in classrooms across the curriculum. Those students who took up the opportunity developed a supportive learning community over the year they were involved in the project and indicated that their awareness of the wider role of mathematical thinking in their courses and in the teaching role was enhanced through their participation in research interviews and the pre/post focus groups.

Role of the website

Students made very little use of the MARKITE website which included information relevant to mathematics across the curriculum, data literacy, administration, and issues profiled in the media (a response to lecturer identification of the political dimension of mathematical thinking), despite school leaders and beginning teachers endorsing the usefulness of a website and the website content.

d. Student teachers' appreciation of mathematical thinking across contexts (Research Question 2)

While we had hoped to monitor the development of individual student thinking over time, honing in on changes from the beginning and end of the year, including pre and post school placement/practicum, it proved difficult to recruit and retain students for this purpose. Incentives such as offering morning tea and lunch did not prove to be particularly successful. The findings reported here are drawn from discussions with 10 students over the course of 2016, the final year of the project, only as illustrative of trends over the other 2 years.

We analysed the discussions using the Bills et al. (2006) notion of example space, which they propose to explain the development of student understanding of number concepts. They define this as the set of examples which a person can access at any one time and the richness of the interconnections between the examples they can access.That is, a person's example space is the collection of examples they associate with a particular concept at a particular time or context. Their proposition is that example spaces are dynamic and evolve. Mason and Goldenberg (2008) note that some parts of a person's example space may be more accessible at a given time than others, with the less accessible parts requiring a trigger which, during group discussions, can be provided by another group member's example. The focus group discussions followed a pattern that was reflective of this as students elaborated on, and made connections to, their peers' contributions. The student examples covered teaching across the curriculum, consideration of student assessment data, and administration tasks.

By the end of the year, focus group students were generally confident that they had developed the breadth of mathematical knowledge they needed as a teacher: Vanya (a pseudonym): "From where I was at the beginning of the year, I'm confident. Scared, you know, but where I am now? A positive attitude, confident, and I have my study buddy. So, it's really nice to share ideas. So I'm sure I'm really good." Another student built on this comment explaining:

> I guess I'm just much more aware of it (mathematical thinking) now. Like if I ask five year olds to line up in twos, or to go into groups of four, they might not know that. Before I started the course, I just thought that would be quite a basic fact. But then even kids higher up in the spectrum still might not have that basic knowledge. So it's almost like the more you learn, the less you know in some respects. ...if I have a game, or I'm teaching in a different curriculum area and I know that their Maths might not be strong then I can come back to that and teach to where Maths is in the class. Yeah. (Richard)
>
> Yeah, I don't really know what to say. At the start of the year, I was a bit sort of apprehensive of teaching Maths. My confidence has grown throughout the year. For me it's similar to what Richard says, assumptions on what children know. So just taking those steps back to think through every step and think how they could solve problems. And to give them time to work out problems. (Lance)

Mason and Goldenberg assert that each time a connection is made it is strengthened and more likely to come to mind in the future. Seen this way the sharing of examples of the use of mathematical thinking in the student teacher discussions served both as an indicator of students' current understandings as well as a catalyst for enhancing their understanding of the role of mathematical thinking in teachers' professional work.

The beginning teacher data also highlighted the importance of making connections over time and showed how links had been made between their ITE course work and their teaching role. They could articulate and provide many examples of how mathematics was used in a teacher's professional role across the curriculum, to make assessment decisions, and in administration.

Implications for practice

There is value in a focus on raising awareness.

Alerting lecturers and student teachers to the possibility of mathematical and statistical thinking in their courses can sensitise them to the range of contexts where this is relevant and informs action. Strategies for raising awareness/embedding would seem to be worthy of further exploration when developing tertiary programmes.

The development of a curriculum map, use of a mapping template, and a curriculum map itself are useful tools for exploring and making explicit cross programme ideas.

The map and mapping template acting as boundary objects facilitated communication and cooperation across different groups and communities. This played a valuable role in expanding and clarifying what mathematical thinking is and in identifying opportunities for learning related to mathematical thinking in courses.

Giving value to regular conversation about cross programme ideas can enhance lecturer understanding of these ideas and help develop the shared understanding and vision that is essential for substantial and systematic change.

Conversations about mathematical thinking, whether centred on completing the template, reflecting on the map, or more exploratory, served to stimulate thinking amongst lecturers about what mathematical thinking there was in their courses. It also prompted them to consider how, when, and why they might highlight for students the embedded mathematical thinking. Lecturers who participated reported they found the conversations valuable for their own practice.

Involvement in research can stimulate and support change.

The research interview process itself can be viewed as an intervention. In our case, the interviews served to value and make visible mathematical thinking as an aspect of all coursework in a programme and hence as part of all lecturers' work in much the same way it is an influence across the breadth of teachers' work.

The interviews served a similar function for students. Participant students enjoyed the pre and post practicum discussions and their commentary confirmed that their example space of instances of the use of mathematical thinking across a teacher's role expanded over time. This indicated an increased awareness and, importantly, the students linked this awareness to actions they might take. The notion of raising awareness as a strategy that includes repeated focused discussion coupled with greater lecturer attention holds promise as a means for fostering the learning of concepts that are cross-curricular. It highlights the role of using contexts that students are familiar with or have an investment in when offering support.

Programmes need to provide multiple opportunities for learning and using ideas as well as feedback on their use in action as part of developing student teachers' self-critical and reflective practice.

Student teachers' self-assessment and confidence did not correlate with their answers being correct in the pre-test. This poses a challenge for mathematics educators and the ITE programme as a whole. Programmes need to provide multiple opportunities for students to encounter, use, and evaluate their understanding of an idea and develop a reflective/ critical/questioning stance through collaborative practice.

There remain challenges in designing programmes that effectively encourage and assist student teachers to engage with self-regulated resources even when these are made available. Developing student teachers' adaptive help seeking is complex.

The lack of use of the dedicated website, which was available during the second and third years of the project, was disappointing. It seems that students are hesitant to approach people they do not know or with whom they have not developed a productive working relationship. These findings raise questions about how and why students access self-regulated resources and how we can develop the adaptive help seeking dispositions required of teachers. We might usefully ask: Is there sufficient time in our programmes for student teachers to identify content and skill weaknesses and develop strategies to address these?

The importance of personal relationships and the need for high-level leadership endorsement of projects that aim to work across programmes.

As with any initiative, time and effort and mutual trust and respect are required to develop shared goals and vision. Our experience of the challenge and complexity of recruiting lecturers and students into the project reaffirmed the importance of personal relationships and the need for high-level leadership endorsement of projects that aim to work across programmes.

Successful strategies in the project were all built on relationships. Lecturers built trusting and purposeful relationships with researchers and were prepared to deprivatise their practice. Over the 3 years, conversations and consequently data collected became richer and reflected that lecturers were understanding the purpose of the project and adopting practices to support the intent. Student teachers participated in ongoing focus groups because they built relationships with the researchers and saw value in participating in the activities.

Challenges and limitations of the project

Conceptual framing

One of the most pervasive challenges was that of helping participants appreciate that our focus was not on mathematics education but on the mathematical thinking that was required across the breadth of a teacher's role. We needed to keep reinforcing with programme lecturers that all had something to contribute to the project. We also needed to reassure students that we were not focused on their mathematical understanding per se but rather on how they understood and used this in their teacher role. This challenge highlighted the extent to which learning areas can become siloed; the project illustrated the value of breaching these silos.

Development of useful assessment items

The development of an assessment to determine how student teacher knowledge, attitudes, and critical awareness changed over time was more complex than originally conceptualised. Selecting appropriate items proved difficult, particularly in the knowledge area. There was much debate about the purposes of the assessment and the types of items that would give us appropriate information. For example, we debated if we should select items to show conceptual understanding or items that were based on actual teaching contexts which students could be unfamiliar with. We were aware that many students are anxious about mathematics and assessing them at the start of a course could negatively impact or reinforce their perceptions that they were not good at mathematics. As a result, even though the assessment was modified each year we still think this needs to be refined before it is shared with other institutions. Interestingly, when the assessment was trialled with a range of groups in the faculty (undergraduate and Master's programmes) there was very little difference between or within groups.

Lecturer participation

The cross-programme MARKITE design assumed that course staffing would be reasonably stable across the 3 years. However, there was a large number of unanticipated lecturer changes leading to a lack of cumulative development in understanding and practice in some areas. Where individuals taught a course for the 3 years, and willingly participated in the project activities, we saw modifications in their course content and activities to highlight mathematical thinking. If there was a significant gap between repeat delivery, the impact was not as great. Nonetheless, the development of mapping templates and regular meetings to establish shared understandings across a programme are being used in other programmes where we are attempting to embed concepts across it as they are reviewed.

Student participation

While we were aware that recruiting students would be challenging, the process was even more difficult than we had expected. A number of students each year expressed that they were anxious about mathematics and hence they were reluctant to complete the assessments and participate in follow-up interviews, mentoring, and post testing. Over the course of the project, we employed a number of strategies to recruit and retain students such as varying the method of communication and offering morning tea or lunch but, overall, our experience affirmed the importance of relationships. When the research team had an established relationship with a student group, students were more likely to participate.

Self-report data

The study findings are based largely on interview data. In the case of lecturers, however, their commentary was supported by analysis of their course outline and to examples from practice and verification from students. For students, probing of the examples that were shared during the focus groups and for some the knowledge of their coursework by the researcher and knowledge of the school practicum context gave us confidence that the examples were well grounded in experience.

Development of the website

This was more challenging and slower than anticipated. We initially worked with a web designer to produce a site, which had capacity to be both a repository and to be interactive. We developed a functioning website populated with quality assured resources, websites, and application links by the end of the first year. The website development included the provision of tools to monitor engagement and use of resources in the hub. Lecturers engaged with the material and gave us feedback. In May of year two, the site was hacked and crashed. Our IT support team recovered the site material and resolved any possible ongoing issues by turning off some of the tools on the site and we employed a graduate to work on the site. Issues such as copyright needed to be addressed prior to the website being made live. As a result the website was only accessible from within the university and we were unable to share more widely. Google analytics built into the site showed very little engagement despite the site being endorsed by school leaders and student teachers.

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Project publications

Journal articles

Cooper, B., Furness, J., & Cowie, B. (Submitted). Curriculum maps and mapping to support boundary encounters: Enhancing the visibility of mathematical thinking in initial teacher education. *Journal of Teacher Education.*
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Practitioner community outputs

Cooper, B., & Cowie, B. (2014). Mathematical thinking and reasoning proficiency in primary teacher education programmes: MARKITE. In *TEFANZ: Spicing up Teacher Education, Spaces, Places, Innovations and Connections: Conference Proceedings* (pp. 19–20). Hamilton: Teacher Education Forum of Aotearoa New Zealand (TEFANZ).

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Research community outputs

Bailey, J., Peter, M., Taylor, M., Cowie, B., Cooper, B. C., & Furness, J. (2015, November 17–20). *Student-teacher perceptions of the role of mathematical and statistical thinking in teachers' work.* Paper presented at the Emancipation through Education, NZARE conference, Te Whare Wānanga o Awanuiārangi, Whakatāne, New Zealand.

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